***Applied Mathematics-2***

***Paper Code: BS-112***

***Assignment-2***

## *Unit-1*

Ques1: - Prove that sinh z is analytic and find its derivative.

Que2: - Determine p such that the function $f(z) = \frac{1}{2}\log(x^2 + y^2) + itan^{-1}\frac{px}{y}$ is analytic.

Que3: - Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.

Que4: - Find all the roots of the equation sin z = cosh 4.

Que5: - Prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i) - y^2(1-i)}{x^2+y^2}$, $(z \neq 0), f(0) = 0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

Que6: The function $f(z) = \frac{1}{1-e^z}$ has what type of singularity?

Que7: - Evaluate $\int_C \frac{dz}{z^2}$, where $C: |z| = 1$ .

Que8: - Evaluate $\int_C \frac{z^4\, dz}{(z+1)(z-i)^2}$ where C is the ellipse $9x^2 + 4y^2 = 36$ **.**

Que9**: -** State and proof Cauchy integral formulae.

Que10: - Evaluate $-\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where $c: |Z - i| = 2$.

## Unit-2

Que1: State residue theorem and use it to evaluate $\int_C \frac{dz}{z^8(z+4)}$ where C is the circle **8**

(i) $|z| = 2$
(ii) (ii) $|z + 2| = 3$.

Que2: Use convolution theorem, to evaluate $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

Que3: Show that the transformation $w = \frac{i-z}{i+z}$ maps the real axis of the z-plane onto the circle $|w| = 1$, and the half plane $y > 0$ onto the interior of the unit circle $|w| < 1$ in the $w$-plane. Also obtain the condition under which the transformation $w = \frac{az+b}{cz+d}$ maps a straight line of $z$-plane into a unit circle of $w$-plane.

Que4: Expand the function $\frac{e^{2z}}{(z-1)^3}$ about $z = 1$ in Laurent's series.

Que5: Evaluate $\int_0^\infty \frac{\cos ax}{x^2+1} dx.$

Que6: Prove that $\int_0^\infty \frac{dx}{x^6+1} = \frac{\pi}{3}$

Que7: Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta+a^2} dx.$

Que8: Evaluate $\int_0^\infty \frac{\sin ax}{x} dx, a > 0$

Que9: Find the linear fractional transformation that maps $z_1 = -1, z_2 = 0, z_3 = 1$ and $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.

Que 10: - Find the points at which w=sin z is not conformal.

**Unit-3**

Que1: Find the inverse Laplace transform of $\frac{s}{s^4+4a^4}$ .

Que2: Expand $f(x) = x^2$ in a Fourier sine series in $0 < x < 1$.

Que3: A particle moves in a line so that its displacement $x$ from a fixed-point O at any time $t$, is given by $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80\sin 5t$. If initially particle is at rest at $x = 0$, find its displacement at any time $t$.

Que4: Find the Fourier series for the function $f(x) = e^{ax}$ in the interval $(-\pi, \pi)$.

Que5: Find the Laplace transform of **8**

(i). $f(t) = \begin{cases} \sin(t-a), & t > a \\ 0, & t < a \end{cases}$

(ii). $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt$

Que6: Solve the initial value problem $y'' + 3y' + 2y = H(t-\pi)\sin \quad 2t$, $y(0) = 1, y'(0) = 0.$

Que7. Find the Laplace transform of $te^{-t}\sin 3t$.

Que8. State and prove second shifting theorem.

Que9. Find the cosine and sine transform of $f(x) = 2e^{-5x} + 5e^{-2x}$.

Que10. Find the inverse Fourier transform of $\frac{1}{30+11i\omega-\omega^2}$.

**Unit-4**

Que1: Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(x,0) = 6e^{-3x}$.

Que2: A string is stretched and fastened to two points l apart. Motion is started by displacing the string $y = a\sin\frac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$.

Que3: Eliminate the arbitrary function 'f 'from the equation $z = f(\frac{xy}{z})$.

Ques4: Use the method of separation of variable to solve PDE $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ , $u(x,0) = 3e^{-4x}$.

Que5: A particle moves in a line so that its displacement $x$ from a fixed-point O at any time $t$, is given by $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80\sin 5t$. If initially particle is at rest at $x = 0$, find its displacement at any time $t$.

*Applied Mathematics-2*

*Paper Code: BS-112*

*Assignment-1*

## *Unit-1*

**Ques1**: - Test the differentiability of the function $f(z) = \frac{(\bar{z})^2}{z}$ for $z \neq 0$ and $f(0) = 0$.

**Ques2** :- Discuss the analyticity of the function $f(z) = z\,\bar{z}$

**Ques3**:- Find an analytic function $f(z) = u(r, \emptyset) + iv(r, \emptyset)$in term of z such that $v(r, \emptyset) = r^2 \cos 2\emptyset - r\cos\emptyset + 2$.

**Ques4** :- Find the analytic function $f(z) = u + iv$ if $u + v = (x + y)(2 - 4xy + x^2 + y^2)$.

**Ques5**:- Find the image of the infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Also show the region graphically.

**Ques6** :- Evaluate $\int_C^{1+i} (x^2 - iy)\, dz$ along the path $y = x$.

**Ques7** :- Evaluate $\int_C \frac{dz}{z^2}$, where $C: |z| = 1$.

**Ques8** :- Evaluate $\int_C \frac{3z^2+z}{z^2-1} dz$ where $C: |z - 1| = 1$.

**Ques9** :- State and prove Cauchy's internal formula and evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C:$|z - i| = 2$.

**Ques10** :- Obtain the Taylor's expansion of $f(z) = \frac{1}{z^2+(1+2i)z+2i}$ about z=0.

**Ques11** :- Find the nature and location of singularities of $f(z) = (\mathrm{z} + 1)\sin\left(\frac{1}{z-2}\right)$.

## *Unit-2*

**Ques1:** Expand $\frac{e^{2z}}{(z-1)^3}$ about $z = 1$ in Laurent's series.

**Ques2:** Find the nature and location of the singularity of the function $f(z) = z^2 e^{1/z}$.Hence find the residue of $f(z)$ at its pole inside the circle $|z| = 2$.

**Ques3:** Determine the poles and residue there for the function $z \cos\left(\frac{1}{z}\right)$.

**Ques4.** Using the residue theorem, Evaluate $\oint_C \frac{dz}{z^4+1}$ Where C is the circle $x^2 + y^2 = 2x$.

**Ques5** :- Evaluate $\int_C \frac{e^z\, dz}{(z+1)^2(z-2)}$ ,where $C: |z| = 3$.

**Ques6** :- Evaluate $\int_0^{2\pi} \frac{d\,\theta}{1-2a\, sin\theta+a^2}$ .

**Ques7** :- Evaluate $\int_0^{\infty} \frac{dx}{x^4+1}$.

**Ques8** :- Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)^2} dx$.

**Ques9** :- Show that the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into the circle of radius unity in $w$-plane and find the centre of the circle.

**Ques10** :- Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path y=x.

***Unit-3***

**Ques1** :- Find the Laplace transform of $L(\sinh at \cos at)$.

**Ques2** :- Evaluate $L(\sin h^3\, 2t)$.

**Ques3** :- Find the inverse Laplace transform of $\frac{s^2+2s-3}{s(s-3)(s+2)}$ .

**Ques4** :- Use convolution to find $L^{-1} \frac{s}{(s^2+a^2)^3}$.

**Ques5** :- Use convolution theorem to evaluate $\frac{s^2}{(s^2+w^2)^2}$.

**Ques6 :-** Find the solution of the initial value problem $y'' + 4y' + 4y = 12t^2e^{-2t}, y(0) = 2\ and\ y'(0) = 1$.

**Ques7 :-** Express the function $f(t) = \begin{matrix} 2t & for\ 0 < t < 5 \\ 10 & for\ \ t > 5 \end{matrix}$

**Ques8:-** Find the inverse Laplace transform of

$$\frac{e^{-\pi s}}{s^2 + 4}$$

**Ques9:-** Find the Fourier sine transform of $\frac{1}{x}$.

**Ques10:-** Use Fourier transform to solve the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}\ ; 0 < x < \infty,\ t > 0 \text{ subject to the condition}$$

$$u(0,t) = 0, t > 0$$

***Unit -4***

**Que1**: Solve $u_{xy} = -u_x$.

**Que2**: Derive wave equation by using D'Alembert.

**Que3**. Find the temperature u(x , t) in a laterally insulated copper bar 80 cm long if the initial temperature is 100 sin($\frac{\pi x}{80}$) ℃ and the ends are kept at 0 ℃ .How long will it take for the maximum temperature in the bar to drop to 50℃ ? First guess, then calculate. Physical data for copper: density 8.92 g / $cm^3$ specific heat 0.092 cal (g ℃), thermal conductivity 0.95 cal/ (cm sec ℃).

**Que4**. The technicalities encountered in cases can often be avoided. For instance, find the potential inside the sphere S; r = R = 1 when S is kept at the potential $f(\varphi) = \cos 2\varphi$. (Can you see the potential on S? What is it at the North Pole? The equator? The south pole?)

**Que5.** A tightly stretched unit square membrane starts vibrating from rest and its initial displacement is $k\sin 2\pi x \sin \pi y$. Show that deflection at any instant is $k\sin 2\pi x \sin \pi y \cos(\sqrt{5}\pi ct)$.

Assignment-4
B.Tech, Semester-II
Paper Code: BS-112
Applied Mathematics-2

## Unit-I

Q1. If $z1 = 2 + 8i$ and $z2 = 1 - i$, then find $|\frac{z1}{z2}|$.

Q2. Evaluate $(cos(\frac{\pi}{9}) + isin(\frac{\pi}{9}))^{18}$.

Q3. Define an analytic function. Check whether $f(z) = \frac{1}{z^3}$ is analytic or not?.

Q4. Find the analytic function $f(z)$ of which real part is $u = e^x(xcosy - ysiny)$.

Q5. Evaluate the integral $\int_c \frac{sin\pi z^2 + cos\pi z^2 dz}{(z-1)(z-2)}$ where $|z| = 3$.

Q6. Expand $f(z) = z^3 - 10z^2 + 6$ at $z = 3$ using Taylor Series.

Q7. Explain different types of singularities and determine the nature of $f(z) = \frac{z^2}{z+2}$.

Q8. Evaluate the integral $\int_0^{1+i}(x^2 - iy)dz$, (i) along the straight line $y = x$, (ii) along the curve $y = x^2$.

Q9. What kind of singularity does $f(z) = \frac{e^z+1}{e^z-1}$ has at $z = 0$.

Q10. Evaluate $\int_c \frac{dz}{z^2+9}$ where (i) 3i lies inside c, (ii) $\pm 3i$ lies inside c.

## Unit-II

Q1. Expand $f(z) = \frac{1}{(z+2)(z+3)}$ in the following region (a) $1 < |z| < 3$, (b) $0 < |z + 1| < 2$.

Q2. Determine the residue of $\int_c \frac{z^2 dz}{(z-1)^2(z+2)}$ where $c = |z| = 5/2$.

Q3. Find the image of the infinite strip $1/4 \leq y \leq 1/2$ under the transformation $\omega = 1/z$. Show the region graphically.

Q4. Find the bilinear transformation which maps the points $2, i, -2$ of z-plane to the points $1, i, -1$ respectively of $\omega$ plane.

Q5. Find the image of the region $1/2 \le x \le 1$ under the transformation $\omega = z^2$.

Q6. Find the expansion of $f(z) = \frac{1}{z-z^3}$ in the region $1 < |z-1| < 2$.

Q7. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.

Q8. Find the coefficient of $z^2$ in the expression of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.

Q9. Calculate the residue of the function $f(z) = \frac{1}{(z-4)(z+1)^3}$.

Q10. Find the value of the integral $\int_c (z-2)^3 dz$ where $c$ is the circle $|z-2| = 4$.

## Unit-III

Q1. State first shifting theorem. Hence find the laplace transform of

$$f(t) = \begin{cases} 2+t^2, & \text{if } 0 < t < 2 \\ 6, & \text{if } 2 < t < 3 \\ 2t-5, & \text{if } 3 < t < \infty \end{cases}$$

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Q2. Find the laplace transform of (i)$t^2 e^{-2t}$, $\quad (ii) \sin at \sin bt$.

Q3. Find the fourier series for $f(x) = e^{ax}$ in $(0, 2\pi)$.

Q4. Find the laplace transform of

$$f(t) = \begin{cases} \cos(t - 2\pi/3), & \text{if } t > 2\pi/3 \\ 0, & \text{if } t < 2\pi/3 \end{cases}$$

Q5. Find $L^{-1}[\frac{3}{s^2} + \frac{2}{s^2+9}]$.

Q6. Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$ given $x(0) = 1, x(\pi/2) = -1$ using laplace transform.

Q7. Solve the integral equation using convolution theorem $y(t) = t + \int_0^t y(x) \sin(t-x) dx$.

Q8. Find the laplace transform of $\int_0^t \frac{e^t \sin t}{t}$.

Q9. Find the inverse Fourier transform of $X(\omega) = \frac{6+4(j\omega)}{(j\omega)^2+6(j\omega)+8}$.

Q10. Find the Fourier series solution to the differential equation $y'' + 2y = 3x$ with the boundary conditions $y(0) = y(1) = 0$.

## Unit-IV

Q1. A tightly stretched string with fixed end points$x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating by given to each of its points a velocity $\lambda(l-x)$, find the displacement of the string at any distance x from one end at any time t.

Q2. A rod of length l with insulated sides is initially at a uniform temperature $u_0$. Its ends are suddenly cooled at $0°C$ and are kept at that temperature. Find the temperature function u(x,t).

Q3. Find the deflection u(x,y,t) of a rectangular membrane $(0 \leq x \leq a, 0 \leq y \leq b)$ whose boundary is fixed , given that it starts from rest and $u(x, y, 0) = xy(a - x)(b - y)$.

Q4. Solve the vibrating string problem with$u(0, t) = 0 = u(l, t)$

Q5. What is geometrical meaning of $\frac{\partial z}{\partial x} and \frac{\partial z}{\partial x}$ ?

# Assignment-3
# B.Tech, Semester-II
# Paper Code: BS-112
# Applied Mathematics-2

## Unit-I

Q1. Suppose $z = (2-i)2 + \frac{(7-4i)}{(2+i)} - 8$, express $z$ in the form of $x + iy$ such that x and y are real numbers.

Q2. Evaluate $(cos(\frac{\pi}{16}) + isin(\frac{\pi}{16}))^8$.

Q3. Split $Sinh(x+iy)$ and $Cosh(x+iy)$ into real and imaginary parts.

Q4. Check if $\lim_{z\to 0}(\frac{z}{\bar{z}})^2$ exists or not?.

Q5. Check whether $f(z) = e^{-\bar{z}}$ is analytic or not?.

Q6. Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find conjugate harmonic function $v$ of $u$.

Q7. Evaluate the integral $\frac{1}{2\pi i}\int_c \frac{ze^z dz}{(z-a)^3}$ where the point $a$ lies within the closed curve $c$.

Q8. Expand $\frac{1}{1+z}$ about $z = 1$ by Taylor series.

Q9. What is the behaviour of $f(z) = Cot(\frac{1}{z})$ at $z = 0$.

Q10. Explain order of a pole. Determine the poles in the following (i) $\frac{1-e^{2z}}{z^4}$, $(ii)\frac{z^2}{(z-1)(z-2)^2}$.

## Unit-II

Q1. Expand $f(z) = \frac{1}{(z+2)(z+3)}$ in the following region (a) $|z| > 3$, (b) $|z| < 1$.

Q2. Determine the residue of $\int_c \frac{dz}{z(z+4)}$ where $c = |z+2| = 3$.

Q3. Find the image of circle $x^2 + y^2 = 4y$ under the transformation $1/z$.

Q4. Find the bilinear transformation which maps the points $\infty, i, 0$ of z-plane to the points $0, i, \infty$ respectively of $\omega$ plane.

Q5. Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta+a^2}$, $a^2 < 1$.

Q6. Find the image of z-plane bounded by $x = 0, y = 0, x = 1, y = 2$ under the transformation $\omega = ze^{i\pi/4}$.

Q7.Find a Linear fractional transformation with the property $f(1) = 1 + i, f(i) = 1 - i, f(-i) = 2$.

Q8. Find the coefficient of $\frac{1}{z-1}$ in the laurent series expansion of $\frac{1}{(z-1)(z-2)}$ in the region $0 < |z - 1| < 1$.

Q9. Solve $\int_0^\pi \sin^2\theta \cos^4\theta d\theta$.

Q10. Calculate the residues of the function $f(z) = \frac{1}{(z-4)(z+1)^3}$.

# Unit-III

Q1. State first shifting theorem. Hence find the laplace transform of $\sinh at \cos at$

Q2. Find the laplace transform of (i)$\frac{1-\cos t}{t^2}$, $(ii) \sin^3 2t$.

Q3. Find the laplace transform of $\int_0^t \frac{\cos at - \cos bt}{t}$.

Q4. Find the laplace transform of

$$f(t) = \begin{cases} e^{-4(t-3)} \sin 3(t-3), & \text{if } t > 3 \\ 0, & \text{if } t < 3 \end{cases}$$

Q5. Find $L^{-1}[\frac{3s+4}{s^2+16} + \frac{2}{s+3}]$.

Q6. Using convolution theorem evaluate $L^{-1}[\frac{s^2}{(s^2+a^2)(s^2+b^2)}]$.

Q7. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$ given that $y(0) = 1, y'(0) = 0, y''(0) = -2$ using laplace transform.

Q8. Find fourier series for $f(x) = x \sin x$ where $-\pi < x < \pi$.

Q9. Find the inverse Fourier transform of $X(\omega) = \frac{1+3(j\omega)}{(3+j\omega)^2}$.

Q10. Find the solution of the ODE, $\frac{-d^2u}{dx^2} + a^2u = f(x)$, $-\infty < x < \infty$ using Fourier transformation method.

# Unit-IV

Q1.Solve the equation

$$\frac{\partial^2 z}{\partial^2 x} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

by the method of separation of variables.

Q2. The points of trisection of a string are pulled aside through the same distance on opposite side of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the midpoint of the string always remains at rest.

Q3. An insulated rod of length l has its end A and B maintained at $0°C$ and $100°C$ respectively until steady state conditions prevail. If B is suddenly reduced to $0°C$ and maintained at $0°C$, find the temperature at a distance x from A at time t.

Q4. Solve the heat problem by the method of convolution.

Q5. Derive Solution of Laplace's equation in cylindrical coordinates.