Applied Mathematics-2

Paper Code: BS-112

Question Bank

Unit-1

Ques1: - Show that $\mathrm{f}(z) = \bar{z}$ has no derivative.

Ques2: - Show that the function ln z is analytic for all z except when $Rez \leq 0$.

Ques3: - If $u(r, \emptyset) = \left(r - \frac{1}{r}\right) \sin \emptyset, r \neq 0$ find an analytic function $f(z) = u + iv$.

Ques4: - Find the line of the force of the gravitational field whose equipotential lines are $e^{x}(x \cos \mathrm{y} - \mathrm{y} \sin \mathrm{y}) = C$. Also, find the complex potential.

Ques5: - Under the mapping, $w = ze^{ip/4}$. Find the region in $w - plane$ corresponding to the triangular region on the $z - plane$ bounded by the lines $x = 0, \mathrm{y} = 0$ and $x + \mathrm{y} = 1$.

Ques6: - Evaluate $\int_C (z-a)^n \, dz$ where C is the circle with centre $'a'$ and radius r.

Also discuss the case when $n = -1$, that is evaluate $\int_C \frac{dz}{z-a}$.

Ques7: - Evaluate $\int_C z^2 \, dz$.

Ques8: - Evaluate $\int_C \frac{5z-2}{z^2-a} dz$ where $C: |z| = 2$.

Ques9: - Use Cauchy's internal formula to evaluate $\oint_C \frac{(\sin \pi z^2 + \cos \pi z^2)}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$.

Ques10: - Find the Taylor's series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ about the circle with centre at the origin.

Ques11: - Find the type of singularity of the function $f(z) = \frac{1-e^{2z}}{z^3}$ at z=0.

Unit-2

Ques1: Expand the function $\frac{1-\cos z}{z^3}$ in Laurent's series about the point $z = 0$.

Ques2: Determine the poles of the function $f(z) = \frac{z^2}{(z-1)(z-2)^2}$ and residue at each pole.

Ques3: Determine the poles and residue there for the function $\frac{z}{\cos z}$.

Ques4: State residue theorem and use it to evaluate

$\int_C \frac{dz}{z^8(z+4)}$ where C is the circle (i) $|z| = 2$ (ii) $|z+2| = 3$.

Ques5: - Evaluate the complex integral $\int_C \frac{z^2\, dz}{(z-1)^2(z+2)}$,where $C: |z| = 3$.

Ques6: - Evaluate $\int_0^{2\pi} \frac{d\,\theta}{1-2a\, cos\theta+a^2}$,where $a^2 < 1$.

Ques7: - Apply residue theorem to evaluate $\int_0^{\infty} \frac{\cos ax}{x^2+1} dx$

Ques8: - Evaluate $\int_0^{\infty} \frac{\sin ax}{x} dx\,, a > 0$.

Ques9: - Show that the mapping $w = e^z$ is conformal in whole of the z-plane.

Unit-3

Ques1: - Find the Laplace transform of $e^{-2t}(3\, cos4t - 2\, sin5t)$.

Ques2: - Using Laplace transform, evaluate $\int_0^{\infty} t^3 e^{-t} \sin t\;\, dx$.

Ques3: - Find the inverse Laplace transform of $\frac{s+4}{s(s-2)(s^2+4)}$.

Ques4: - Use convolution to find $L^{-1} \frac{1}{(s^2+a^2)^2}$.

Ques5: - Use convolution theorem to evaluate $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

Ques6: - Use Laplace transform to solve the equation $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 8y = 1$ given that $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$

Ques7: - Find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

Ques8: - Find the inverse Laplace transform of

$$\frac{e^{-\pi s}}{s^2}$$

Ques9: - Find the complex Fourier transform of direct delta function $\delta(t-a)$.

Ques10: - Calculate the inverse Fourier transform of $\frac{1}{(4+w^2)(9+w^2)}$ using convolution theorem.

Unit -4

Ques1: - Solve $u_{xx} - u = 0$.

Ques2: - Solve wave equation by using separation of variable.

Ques3: - Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$f(x) = \begin{cases} x, & 0 < x < L/2 \\ L-x, & L/2 < x < L \end{cases}$$

Ques4: - Find the temperature in the infinite bar if the initial temperature is

$$f(x) = \begin{cases} U_0 = const, & |x| < 1 \\ L-x, & |x| > 1 \end{cases}$$

Ques5: - Find the differential equation of all spheres whose centers lie on z-axis.

Ques6: - Use the method of separation of variables to solve the partial differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, $u(x,0) = 4e^{-x}$.

Ques7: - Find the Fourier transform of xe^{-ax^2} , $a > 0$.

Ques8: - Determine the Fourier sine integral of the function

$$f(x) = \begin{cases} x & ,0 < x < 1 \\ 2-x & ,1 < x < 2 \\ 0 & ,x > 2 \end{cases}$$