GURU TEGH BAHADUR INSTITUTE OF TECHNOLOGY

NEW DELHI

PROBABILY, STATISTICS AND LINEAR ALGEBRA

AIML/DS-209

PRACTICE QUESTIONS

Unit 1

1. Define Sample Space and Event. What is the relation between these two?

2. Given events A and B in a sample space, if $P(A)=0.6$ and $P(B|A)=0.4$, calculate $P(A\cap B)$.

3. A bag contains 5 red and 3 blue balls. If two balls are drawn without replacement, what is the probability of getting one red and one blue ball?

4. In a deck of 52 cards, what is the probability of drawing an ace and then drawing a king without replacement?

5. You have a standard deck of 52 cards. If you draw 5 cards at random, what is the probability that all 5 cards are of the same suit?

6. Two six-sided dice are rolled. What is the probability that the sum of the two numbers is 7, given that at least one of the dice shows a 4?

7. A continuous random variable X has a pdf $f(x) = 3x^2$, $0 \le x \le 1$. Find a and b such that $P(X \le a) = P(X > a)$. $P(X > b) = 0.05$.

8. For the distribution defined by the pdf $f(x) = \begin{cases} x, & 0 < x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 0, & otherwise \end{cases}$. Compute the 'r'th moment about the origin. Hence deduce the first four moments about mean.

9. Let X be a random variable with pdf $f(x) = \begin{cases} \frac{1}{3}e^{\frac{-x}{3}}, x \ge 1 \\ 0, otherwise \end{cases}$,

   Find (a). P (X > 3) (b) E(X) (c) Var(X).

10. Suppose that the life of an industrial lamp, in thousands of hours, is exponentially distributed with failure rate $\lambda = 1/3$. Find the probability that the lamp will last
    a. Longer than its mean life of 3000 hours.
    b. Between 2000 and 3000 hours
    c. For another 1000 hours given that it is operating after 2500 hours.

11. In a dataset, the mean is 25, and the standard deviation is 5. Apply Chebyshev's Inequality to find the minimum proportion of data that must fall within 15 and 35.

12. In a distribution, the mean is 60, and the standard deviation is 8. Use Chebyshev's Inequality to estimate the proportion of data that falls within 44 and 76.

13. A certain medical test for a disease is known to have a false positive rate of 5% and a false negative rate of 2%. If the disease prevalence in the population is 1%, calculate the probability that a person who tests positive actually has the disease.
14. In a deck of cards, 4 cards are red, and 6 cards are black. If you draw two cards without replacement and the first card drawn is red, calculate the probability that the second card is black.

15. A factory produces light bulbs, and 5% of them are defective. If a quality control test detects with a probability of 90%, what is the probability that a bulb is defective given that it failed the test.

16. Find value of k so that following function is a probability density function

$$f(x) = \frac{k}{1+x^2}\,,\ x\epsilon\mathbb{R}$$

17. Let X be a continuous random variable with PDF $f(x) = \begin{cases} x+\frac{1}{2}, & 0 \le x \le 1 \\ 0, otherwise \end{cases}$

Find E(2X+3).

18. Let X be continuous random variable with probability density function $f(x) = \begin{cases} \frac{3}{x^4}, & x \ge 1 \\ 0, otherwise \end{cases}$. Prove that $Mean = 2\ Variance.$

19. Three urns are there containing white and black balls; first urn has 3 white and 2 black balls, second urn has 2 white and 3 black balls and third urn has 4 white and 1 black balls. Without any biasing one urn is chosen from that one ball is chosen randomly which was white. What is probability that it came from the third urn?

20. Find value of k so that following is probability mass function.

X	0.5	1	1.5	2	2.5	3
P(X)	k	2k	k/2	3k	k	3k/2

21. Calculate the first four central moments for a discrete random variable X with the probability distribution:
P(x=1)=0.3, P(x=2)=0.4, P(x=3)=0.2, P(x=4)=0.1

22. Determine the skewness and kurtosis for a continuous random variable Y with the probability density function:

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(y-3)^2}{2}}$$

23. Evaluate the mean and variance of a binomial distribution with parameters n=8 and p=0.3.
24. For a Poisson distribution with a mean λ=4, calculate the probability $P(X=2)$.

25. The probability that a bomb dropped from a plane will strike the target is 1/5. If six bombs are dropped, find the probability that (i) exactly two will strike the target, (ii) at least two will strike the target.

26. A sortie of 20 aeroplanes is sent on an operational flight. The chances that an aeroplane fails to return is 5%. Find the probability that (i) one plane does not return (ii) at the most 5 planes do not return, and (iii) what is the most probable number of returns?

27. The probability that an entering student will graduate is 0.4. Determine the probability that out of 5 students (a) none (b) one and (c) at least one will graduate.

28. Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys, (b) 5 girls, (e) either 2 or 3.

29. In a bombing action there is 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs are required to be dropped to give a 99% chance or better of completely destroying the target.

30. The frequency of accidents per shift in a factory is as shown in the following table:

Accident per shift	0	1	2	3	4
Frequency	180	92	24	3	1

Calculate the mean number of accidents per shift and the corresponding Poisson distribution and compare with actual observations.

31. Fit a Poisson distribution to the following:

x :	0	1	2	3	4
f(x):	46	38	22	9	1

32. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

33. X is a Poisson variable and it is found that the probability that x = 0 is two-thirds of the probability that X = 1. Find the probability that X = 0 and the probability that X = 3.What is the probability that X exceeds 3.

Normal Distribution

34. For a normally distributed variate with mean 1 and S.D. 3, find the probabilities that (i) $3.43 \leq x \leq 6.19$ (ii)$-1.43 \leq x \leq 6.19$.

35. If z is normally distributed with mean 0 and variance 1, find
(i) $P(z \leq -1.64)$
(ii) z_1 if $P(z \geq z_1) = 0.84$

36. In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Estimate the average marks obtained by the candidates, the minimum pass and distinction marks being 40 and 75 respectively. (Assume the distribution of marks to be normal).

37. A manufacturer of air-mail envelopes knows from experience that the weight of the envelopes is normally distributed with mean 1.95 gm and standard deviation 0.05 gm. About how many envelopes weighing
(i) 2 gm or more
(ii) 2.05 gm or more can be expected in a given packet of 100 envelopes.

38. In an examination taken by 500 candidates, the average and the standard deviation of marks obtained (normally distributed) are 40% and 10%. Find approximately
(i) how many will pass, if 50% is fixed as a minimum?
(ii) what should be the minimum if 350 candidates are to passed.
(iii) how many have scored marks above 60

39. The mean height of 500 students is 151 cm. and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students's heights lie between 120 and 155 cm.

40. The mean and standard deviation of the marks obtained by 1000 students in an examination are respectively 34.4 and 16.5. Assuming the normality of the distribution, find the approximate number of students expected to obtain marks between 30 and 60.

41. Find the correlation co-efficient for the following data

x	1	2	3	4	5
y	2	5	3	8	7

42. Find the correlation co-efficient for the following data

x	78	89	97	69	59	79	68	57
y	125	137	156	112	107	138	123	108

43. Find the co-efficient of correlation between industrial production and export using the following data and comment on the result.

Production (in crore tons)	55	56	58	59	60	60	62
Export (in crore tons)	35	38	38	39	44	43	45

44. Ten people of various heights as under, were requested to read the letters on a car at 25 yards distance. The number of letters correctly read is given below:

Height (in feet):
No. of letters

5.1	5.3	5.6	5.7	5.8	5.9	5.10	5.11	6.0	6.1
11	17	19	14	8	15	20	6	8	12

45. Calculate the rank correlation coefficient from the following data showing ranks of 10 students in two subjects:

Maths	3	8	9	2	7	10	4	6	1

Physics	5	9	10	1	8	7	3	4	2

46. Find the Rank Correlation for the following data:

x	56	42	72	36	63	47	55	49	38
y	147	125	160	118	149	128	150	145	115

Unit 3

47. Fit a straight line to the data points (2, 4), (4, 7), (6, 11). What are the slope and y-intercept?

48. Given the data points (1, 3), (2, 5), (3, 9), fit a second-degree polynomial. What is the equation of the parabola?

49. Consider the data set: (1, 2), (2, 5), (3, 10), (4, 17). Fit a polynomial of degree 3. What is the equation of the curve?

50. A sample of 100 items has a proportion of 0.25. Test whether this proportion is significantly different from 0.3 using a Z-test.

51. Compare the proportions of success in two independent samples: Sample 1 (n1=150, p1=0.6) and Sample 2 (n2=200, p2=0.55). Is there a significant difference?

52. Conduct a one-sample t-test to determine if the mean of a sample (n=30, mean=12, s=3) is significantly different from 10.

53. Test the hypothesis that the mean of Group A (n=25, mean=45, s=8) is equal to the mean of Group B (n=30, mean=50, s=10) using a t-test.

54. A pharmaceutical company claims that the mean time for a new drug to take effect is 12 hours. In a sample of 30 patients, the mean time was found to be 11.5 hours with a standard deviation of 2 hours. Test the hypothesis at a 5% significance level.

55. Compare the average scores of two different teaching methods: Method A (n1=20, mean=75, s1=10) and Method B (n2=25, mean=80, s2=12). Test the hypothesis that there is no difference in the effectiveness of the two methods.

56. Two samples are taken from two different populations. Sample 1 (n1=30, mean=50, s1=8) and Sample 2 (n2=40, mean=48, s2=10). Test whether there is a significant difference in the means of the two populations.

57. A company claims that its new software is faster than the old version. In a test, the old software took an average of 8 seconds (n1=15, s1=2), and the new software took an average of 6 seconds (n2=20, s2=1). Test the hypothesis at a 5% significance level.

58. Compare the variability of test scores between two schools: School X (n1=25, s1=15) and School Y (n2=30, s2=12). Test the hypothesis that the standard deviations are equal.

59. Two different methods are used to measure the heights of plants. Method 1 ($n1=18$, $s1=0.5$) and Method 2 ($n2=20$, $s2=0.8$). Test the hypothesis that the standard deviations of the two methods are equal.

60. A manufacturer claims that a new production process reduces variability. In a sample of 20 products using the old process, the standard deviation was 4 units. With the new process, a sample of 25 products had a standard deviation of 2 units. Test the hypothesis at a 1% significance level.

Unit-4

61. Solve the system of linear equations using Cramer's Rule:

$$3x + 2y + z = 7$$
$$2x - y + 2z = 2$$
$$x + 3y - z = 1$$

62. Solve the following system of 3 equations in 3 variables using Cramer's rule:

$x + y + z = 2$
$2x + y + 3z = 9$
$x - 3y + z = 10$

63. Given a matrix B= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, compute the Singular Value Decomposition of B.

64. Determine whether the set of all 2x2 matrices forms a Euclidean vector space. Justify your answer.

65. Consider the subspace W spanned by the vectors u=[1,2,3], and **v**=[4,5,6]. Find the projection matrix P onto W.

66. Show that the product of two Hermitian matrices is Hermitian if and only if the matrices commute.

67. Given a Unitary matrix U, prove that the columns (or rows) of U form an orthonormal set.

68. Apply the Gram-Schmidt process to orthogonalize the set of vectors:

$$u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

69. Given a matrix $C = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$, compute the LU-Decomposition of C.

70. Discuss the conditions under which LU-Decomposition is guaranteed to exist for a given matrix.