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COMPUTATIONAL METHODS

SEM - 3

ES - 201

UNIT-I

ASSIGNMENT-1

Q1. Roundoff the following numbers correct to four significant figures:

3.26425, 687.543, 4985561, 0.70035, 0.00032217, 18.265101.

Q2. Find the roundoff error in sorting the number 752.6835 using the four-digit mantissa.

Q3. If 0.333 is the approximate value of $\frac{1}{3}$,find absolute, relative and percentage errors.

Q4. Find the difference X=$\sqrt{5.36}$-$\sqrt{5.35}$ and evaluate the relative of the result.

Q5. Roundoff the number the numbers 865250 and 37.46235 to four significant figures and compute $E_a,\ E_r\ and\ E_p.$

Q6. Find absolute error and relative error in $\sqrt{6}+\ \sqrt{7}+\ \sqrt{8}$ correct to 4 significant digits

Q7. Explain different types of errors.

Assignment-2

Q1. The error in the measurement of radius of the sphere is 0.3% what is the permissible error in its surface area?

Q2. Write the statement of mean value theorem.

Q3. Find a root of the equation $x^3 - 4x - 9 = 0$ using bisection method in four stages.

Q4. Find a root of the equation $x = e^{-x}$, correct to three decimal places by secant method.

Q5. Solve the equation $logx = \cos x$ to five decimals by Newton-Raphson Method.

Q6. Explain Secant method. Find root of equation $x^3 - 5x + 1 = 0$ by secant method to correct 3 places of decimal.

Q7. If $u = \frac{4x^2y^3}{z^4}$ and errors in x,y, z be 0.001, find relative maximum error in u if x = y = z = 1.

Assignment-3

Q1. Perform four iterations of the Newton's-Raphson Method to obtain the approximate value of $(14)^{1/2}$ starting with the initial approximation $x_0 = 4$.

Q2. Obtain $\sqrt[5]{12}$, to five places of decimals by Newton's Raphson Method.

Q3. Find the cube root of 10.

Q4. Minimize $f(x) = x^2$ over (-5,15), n=5 by Fibonacci search method.

Q5. Minimize $f(x) = x^2 + \frac{54}{x}$ in interval (0,5), using Fibonacci Search Method.

Assignment-4

Q1. Minimize $f(x) = 4x^3 + x^2 - 7x + 14$ within the interval (0,1) using Golden Section Method.

Q2. Find minimum value of $f(x_1 x_2) = x_1^2 - x_1 x_2 + x_2^2$ using steepest descent method such that $|f(X_{k+1}) - f(X_k)| < 0.05$ taking starting point $X_1 = (1, \frac{1}{2})^T$.

Q3. Use steepest Descent Method

$$f(x_1 x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$$

Starting from the point $X_1 = (0,0)$

Q4. Minimize

$$f(x_1 x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$$

By taking the Starting point as $X_1 = [0,0]^T$ by Newton Method.

Q5. Use Nelder-Mead Method to find minimum of

$f(x, y) = x^2 - 4x + y^2 - y - xy$

Given vertices are $V_1 = (0,0),\ V_2 = (1.2,0)\ and\ V_3 = (0,0.8)$

UNIT-II

ASSIGNMENT-1

Q1. Evaluate the following :

1) $\Delta e^x \log 2x$
2) $\Delta\left(\frac{x^2}{\cos 2x}\right)$

Q2. . Evaluate $\Delta^4[(1-2x)(1-3x)(1-4x)(1-x)]$, where interval of differencing is unity.

Q3. Prove that $\Delta^3 y_3 = \nabla^3 y_6$.

Q4. Form the forward difference table for the function

$$\mathrm{f(x)} = \mathrm{x}^3 - 2\mathrm{x}^2 - 3x - 1 \; for \; x = 0,1,2,3,4$$

Find $\Delta^3 f(x)$ and also show that $\Delta^4 f(x) = 0$

Q5. If for a polynomial, five observations are recorded as :

$$y_1 = -6, y_2 = 22, y_3 = 148, y_4 = 492, find \; y_5.$$

ASSIGNMENT-2

Q1. Use the concept of missing data $find\ y_5\ if$

$$y_1 = -6, y_2 = 22, y_3 = 148, y_4 = 492.$$

Q2. Find the missing values in the following table:

x	0	5	10	15	20	25
$f(x)$	6	?	13	17	22	?

Q3. From the following data, estimate the number of students obtained marks between 40 and 45.

Marks	30–40	40–50	50–60	60–70	70–80
Number of Students	31	42	51	35	31

Q4. Find the cubic polynomial with given set of points:

x	0	1	2	3
$f(x)$	5	6	3	14

Hence, evaluate f(0.5).

Q5. For the given set of values, evaluate $\cos 22^0$ *and* $\cos 73^0$, using suitable interpolation techniques.

x	10°	20°	30°	40°	50°	60°	70°	80°
$\cos x$	0.9848	0.9397	0.8660	0.7660	0.6428	0.5000	0.3420	0.1737

ASSIGNMENT-3

Q1. Estimate f(2) from the following data, using Newton's divided difference method.

x	0	1	3	6
y	1	3	55	343

Q2. Find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182, \log_{10} 659 = 2.8189, \log_{10} 661 = 2.8202.$

Q3. Find the polynomial of the lowest degree which assumes the values 1,27,64 for x taking the values 1,3,4 respectively, using Lagrange's interpolation formula and hence find f(2).

Q4. Estimate f(7) from the following data, using Lagrange's interpolation formula:

x	5	6	9	11
y	12	13	14	16

Q5. Find the 4^{th} order divided differences from the given data:

x	0.5	1.5	3.0	5.0	6.5	8.0
y	1.625	5.875	31	131	282.125	521

ASSIGNMENT-4

Q1. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places.

Q2. Find the value of $\int_0^1 \frac{dx}{1+x^2}$ by Gauss's formula for n=2,4.

Q3. Use Romberg's method to compute $\int_0^{\pi/2} \sin x \ dx$ correct to five decimal places.

Q4. Compute $\int_5^{12} \frac{dx}{x}$ using 3-point Gauss Quadrature formula.

Q5. Write the assumptions for interpolation.

UNIT-III

Assignment-1

Q1. Test for consistency and solve the system

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

Q2. Show that equations

$$2x + y = -11$$
$$6x - 20y - 6z = -3$$
$$6y - 18z = -3$$

are not consistent.

Q3. For what value of k the equations

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

have a solution and solve them completely in each case?

Q4. Solve the following system by Gauss's Elimination Method.

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

Q5. Solve the system of equations

$$x + y + z = 6$$

$$3x + 3y + 4z = 20$$

$$2x + y + 3z = 13$$

Using Gauss elimination method with partial pivoting.

Assignment-2

Q1. Apply Gauss -Jordan method to solve:

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

Q2. Solve the following equations by Gauss Jordan Method

$$x + 2y + z - u = -2$$
$$x + y + 3z - 2u = -6$$
$$2x + 3y - z + 2u = 7$$
$$x + y + z + u = 2$$

Q3. Solve the following set of equations by Crout's method

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

Q4. Solve

$$25x + 15y - 5z = 35$$
$$15x + 18y + 0z = 33$$
$$-5x + 0y + 11z = 6$$

Using Cholesky's decomposition, method.

Q5. Find the eigen values and eigen vectors of matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Assignment-3

Q1. Determine the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q2. Obtain by power method, the numerically dominant eigen values and eigen vector of the matrix

$$A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$$

Q3. Find by the Power method the largest eigen value of the matrix
$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Q4. Find the inverse of the matrix $\begin{bmatrix} -2 & 4 & 8 \\ -4 & 18 & -16 \\ -6 & 2 & -20 \end{bmatrix}$ by Doolittle Method.

Q5. Use factorization method, find the inverse of the matrix:

$$A = \begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

Assignment-4

Q1. Determine whether this function is a first-degree spline function

$$S(x) = \begin{cases} x & x\epsilon[-1,0] \\ 1-x & x\epsilon(0,1) \\ 2x-2 & x\epsilon[1,2] \end{cases}$$

Q2. Determine whether the following function is a quadratic spline function

$$Q(x) = \begin{cases} x^2 & -10 \leq x \leq 0 \\ -x^2 & 0 \leq x \leq 1 \\ 1-2x & 1 \leq x \leq 20 \end{cases}$$

Q3. check the given function is cubic splines

$$f(x) = \begin{cases} 5x^3-3x^2 & -1 \leq x \leq 0 \\ -5x^3-3x^2 & 0 \leq x \leq 1 \end{cases}$$

Q4. Calculate the natural cubic spline interpolating the data

X	1	2	3	4
Y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

Take $M_0 = M_3 = 0$

Q5. Obtain the cubic spline approximation for the function defined by the data

X	:	0	1	2
f(x)	:	1	2	33

with $M_0 = 0,\ M_2 = 0$ and hence find f (1.5)

UNIT-IV

ASSIGNMENT-1

Q1. Solve the following initial value problems using Euler's method:

$y' = t + y, y(1) = 0$. Computer $y(1.2)$ using h=0.1.

Q2. Solve the initial value problem

$y' = (t/y), y(0) = 1$

using Euler's method with $h = 0.2$ to get $y(0.2)$. Then, repeat application of the method with $h = 0.1$ to get another estimate of $y(0.2)$. Extrapolate the results assuming that errors are proportional to the step size and compare the result with the exact solution.

Q3. Consider the initial value problem $y' = x(y + x) - 2, y(0) = 2$.
Use Euler's method with step sizes $h = 0.3, h = 0.2$ and $h = 0.15$ to compute approximations to $y(0.6)$ (to 5 decimal places)

Q4. Use the Taylor series method to solve the initial value problem

$u' = t^2 + u^2, u(0) = 1$ for the interval (0 0.4) using two subintervals of length 0.2.

Q5. Apply Taylor series method to integrate $y' = 2t + 3y, y(0) = 1, t \in (0, 0.4)$ with $h = 0.1$

Q6. Solve the following using Picard's method:

(i) $y' = x + y^2, y(0) = 0$. Find $y(0.3)$ to four decimal places.
(ii) $y' = 2 - \frac{y}{x}, y(1) = 2$. Find $y(1.2)$ to four decimal places.
(iii) $y' = y + e^x, y(0) = 0$. Find $y(1)$.
(iv) $y' = x^2 + \frac{y}{2}, y(0) = 1$. Find $y(0.5)$.
(v) $y' = 1 - xy, y(0) = 0$. Find $y(1)$.

Q7. Solve $y' = y - 2x^2 + 1, y(0) = 0.5$ with step size $h = 0.2$ to find $y(1)$ using Modified Euler's method.

ASSIGNMENT-2

Q1. The following IVP is given:

$$y' = 2x + 3y, y(0) = 1$$

Use Taylor's series method to get $y(0.4)$ with step length h=0.1.

Q2. Apply Taylor series method to compute $y(0.2)$ from $y' - 4y = 0$ given that $y(0) = 1$

Q3. Solve by using Picard's method:

$$\frac{dy}{dx} = 2x - y \text{ given that } y=0.9 \text{ at } x=0.$$

Solve for x=0.2, 0.4 and 0.6 and check with exact values.

Q4. Solve $\frac{dy}{dx} = x + y$ with $y(0) = 1$ at $x = 0.0(0.2)0.8$.

Q5. Use Modified Euler's method to obtain $y(0.2)$, $y(0.4)$ and $y(0.6)$ correct to three decimal places given by $\frac{dy}{dx} = y - x^2$ with initial condition $y(0) = 1$.

Q6. Solve $\frac{dy}{dx} = log(x + y), y(0) = 2$ for $y(0.3)$ by RK4 method in two steps.

ASSIGNMENT-3

Q1. Find the value of y when x = 0.1 & 0.2 given y(0) = 1 and $y' = x^2 - y$ using Modified Euler's method.

Q2. Use Picard's method to find the starting values 0.2, 0.4, 0.6,where $y' = x - y^2$ in range $0 \leq x \leq 1 \; s.t. \, y(0) = 0$. Find the solution at x=1 using Milne's method.

Q3. Find the value of y(0.20) for IVP:

$\frac{dy}{dx} = y^2 \sin x \;; y(0) = 1$, using Milne's method taking h=0.05.

Q4. Given $\frac{dy}{dx} = x^2(1 + y)$ and y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979. Evaluate y(1.4).

Q5. Find the value of y(0.20) and y(0.25) from $y' = 2y - y^2$ with y(0)=1, h=0.05.

Q6. Solve IVP : $u' = -2tu^2$, u(0) = 1 with h = 0.2 on the interval [0,0.4]. Use the fourth order Runge-Kutta method and compare with the exact solution.