GURU TEGH BAHADUR INSTITUTE OF TECHNOLOGY

NEW DELHI

COMPUTATIONAL METHODS

AIDS-212

PRACTICE QUESTIONS

1. Use the bisection method to find a root of the function

   $f(x) = x^3 - 6x^2 + 11x - 6$ in the interval [1,3].

2. Find a root of the following equation, using the bisection method correct to three decimal places : $x^3 - 2x - 5 = 0$

3. Find a root of the following equation $x^3 - x^2 - 1 = 0$, using the bisection method correct to three decimal places .

4. Using regula-falsi method, compute the real root of $xe^x = 2$.

5. Using regula-falsi method, compute the real root of $\cos(x) = 3x - 1$.

6. Find the fourth root of 12 correct to three decimal places using the method of false position.

7. Find the negative root of the equation

   $x^3 - 21x + 3500 = 0$ correct to two decimal places by Newton′s method.

8. Using Newton-Raphson method, find a root of the following equation correct to the three decimal places:

   $x^2 + 4 \sin x = 0$

9. Find root of $xe^x - 2 = 0$ correct to the three decimal places by newton Raphson Method.

10. Consider the function $f(x) = x^3 - 6x^2 + 11x - 6$. Use the secant method to find a root of this function. Choose two initial guesses, $x0=2$ and $x1=3$. Perform three iterations of the secant method and provide the estimated root.

11. Apply the secant method to find a root of the function $g(x) = \cos(x) - e^{-x}$ in the interval $[0, \pi]$. Use $x0=1$ and $x1=2$ as initial guesses. Perform four iterations and comment on the convergence behavior.

12. Use the secant method to find the minimum of the function $h(x) = x^2 - 4x + 3$. Choose initial guesses $x0=1$ and $x1=3$. Perform three iterations and report the estimated minimum.

13. Apply the Brent method to find a root of the function $f(x) = x^3 - 6x^2 + 11x - 6$. Choose the interval $[a,b]$ such that $f(a)$ and $f(b)$ have opposite signs. Perform three iterations and provide the estimated root.

14. Use the Brent method to find a root of the function $g(x)=\cos(x)-e^{-x}$ in the interval $[0,\pi]$. Choose the initial guesses $x0=1$ and $x1=2$. Perform four iterations and comment on the convergence behavior.

15. Find Cube root of 16 using Brent Method.

16. Solve the following system of linear equations using Gaussian elimination

$$2x+y-z=5$$
$$4x-2y+3z=10$$
$$-2x+2y+2z=1$$

17. Solve the following system of equation using Gauss elimination method.

3x + 2y + 3z = 18;
2x + y + z = 10;
x + 4y + 9z = 16

18. Solve the following tri-diagonal system with the Thomas algorithm:

10a+2b=12
2a+9b+3c=14
b + 10c +4d =15
3c + 11d = 14

19. Solve the tridiagonal system of equations using a suitable method:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

20. Decompose the matrix A into form LL^T using Cholesky method where

$$A=\begin{bmatrix} 9 & -3 & 3 \\ -3 & 13 & -5 \\ 3 & -5 & 15 \end{bmatrix}$$

21. Solve the system in Ax=b ,A as in Q.20 using Cholesky method by taking $b=[1,-2,3]^T$

22. Solve the following system of equation by using inverse of the coefficient matrix of the system

2x+y-z=8
-3x-y+2z=-11
-2x+y+2z=-3

23. Consider the system of equation

4x-y+z=10

-2x+3y-2z=-5

x+y+z=6

Use the matrix inverse method to solve the system.

24. Using Gauss Seidal method, solve the following set of equations up to 3 decimal places.

3x + y - z = 0,

x + 2y + z = 0,

x - y + 4z = 3

25. Using Gauss Seidal iteration method, solve the following set of equations up to 5 iteration

4x + 2 z = 4,

5x + 2 z = 3,

5x - 4y + 10z = 2

26. Apply Gauss – Seidal iteration method to solve the following equations:

20x+y-2z=17;

3x+20y-z=-18;

2x-3y+20z=25

27. Determine the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

28. Obtain by power method, the numerically dominant eigen values and eigen vector of the matrix

$$A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$$

29. Find by the Power method the largest eigen value of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

30. Determine whether this function is a first-degree spline function

$$S(x) = \begin{cases} x & x\epsilon[-1,0] \\ 1-x & x\epsilon(0,1) \\ 2x-2 & x\epsilon[1,2] \end{cases}$$

31. Determine whether the following function is a quadratic spline function

$$Q(x) = \begin{cases} x^2 & -10 \le x \le 0 \\ -x^2 & 0 \le x \le 1 \\ 1-2x & 1 \le x \le 20 \end{cases}$$

32. Find the polynomial of the lowest degree which assumes the values 1,27,64 for x taking the values 1,3,4 respectively, using Lagrange's interpolation formula and hence find f(2).

33. Estimate f(7) from the following data, using Lagrange's interpolation formula:

x	5	6	9	11
y	12	13	14	16

34. Consider the function $f(x) = e^x \cos(x)$. Use a high-accuracy differentiation formula to approximate the second derivative of f at x=$\frac{\pi}{4}$

35. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places.

36. Find the value of $\int_0^1 \frac{dx}{1+x^2}$ by Gauss's formula for n=2,4.

37. Use Romberg's method to compute $\int_0^{\pi/2} \sin x \ dx$ correct to five decimal places.

38. Compute $\int_5^{12} \frac{dx}{x}$ using 3-point Gauss Quadrature formula.

39. Write the applications of numerical integration.

40. Evaluate $\int_{0.5}^{1.3} \frac{dx}{1+\log x}$ using (i) Trapezoidal rule (ii) Simpson's 1/3 rule (iii) Simpson's 3/8 rule, taking n = 8

41. From the following table, find the area bounded by the curve and x-axis between the ordinates x=7.47 to x=7.52

x	7.47	7.48	7.49	7.50	7.51	7.52
$y = f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

42. The velocity 'v' of an aeroplane which starts from rest is given at fixed intervals of time 't' as shown:

t (minutes)	2	4	6	8	10	12	14	16	18	20
$v = f(t)$ (km/minutes)	8	17	24	28	30	20	12	6	2	0

Estimate the approximate distance covered in 20 minutes.

43. Write the assumptions for interpolation.

44. Write a note on Newton's Cote's Quadrature formula.

45. Using Euler's method, find an approximate value of y corresponding to x = 1, given that dy/dx = x + y and y = 1 when x = 0.

46. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y = 1 at x = 0; find y for x = 0.1 by Euler's method.

47. Apply the Runge-Kutta fourth order method to find an approximate value of y when x = 0.2 given that dy/dx = x + y and y = 1 when x = 0.

48. Using the Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4.

49. Apply the Runge-Kutta method to find the approximate value of y for x = 0.2, in steps of 0.1, if $dy/dx = x + y^2$, y = 1 where x = 0.

50. Define Initial value problems and Boundary value problems.

51. Classify the following PDE's into Hyperbolic, Parabolic and Elliptical

(1) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

(2) $(u_x x + u_y y) = 0$

(3) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (\frac{\partial^2 u}{\partial x \partial y})^2 = x^2 + y^2$